

# Self-force on a scalar point charge in the long throat

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## 1 Introduction

The study of a self-force has a long history. The original investigations focused on the self-acceleration of an electrically-charged point particle in flat spacetime [1]. Later DeWitt, Brehme, and Hobbs [2] studied the influence of the self-force on a charge in a curved spacetime. In contrast to the case of a flat spacetime this force can be nonzero even for a static charge in a curved background. A number of static configurations has been analyzed, including the self-action in the spacetimes of a Schwarzschild black hole [3,4], of a Kerr black hole [5], of a Kerr-Newman black hole [4] and in a spherically symmetric Brans-Dicke field [6]. The analytic approximation of self-force has been obtained for a scalar charge at rest in an axisymmetric spacetime [7]. The self-force can be nonzero for a static particle in flat spacetimes of the topological defects [8]. In curved spacetimes with nontrivial topological structure the investigations of this type have the additional interesting features [9,10].

The effect of self-action is associated with nonlocal structure of the massless field, the source of which is the charged particle. For example, the self-force on a scalar charge is [11]

$$f_\mu = q^2 \left[ \frac{1}{3} (\dot{a}_\mu - a^2 u_\mu) + \frac{1}{6} (R_\mu^\nu u_\nu + R_{\nu\gamma} u^\nu u^\gamma u_\mu) + \frac{1}{12} (6\xi - 1) R u_\mu + \lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\tau-\epsilon} \nabla_\mu G_{ret}(x, x') d\tau' \right] \quad (1)$$

where  $u_\mu$  is the 4-velocity of the particle,  $a_\mu$  is the 4-acceleration,  $\dot{a}_\mu = \partial a_\mu / \partial \tau$  is the derivative of the 4-acceleration with respect to proper time  $\tau$  of a charged

particle,  $G_{ret}(x, x')$  is the retarded scalar Green's function and  $\xi$  is the coupling to the background scalar curvature.

Are there the situations in which the effect of self-action is determined by the local geometry of the curved spacetime? As it is demonstrated below such a situation for the static scalar charge takes place, for example, in the throat of the wormhole if the length of this throat is much more than the radius of throat. As the examples of such wormholes one can consider the spacetimes with metric

$$ds^2 = -dt^2 + d\rho^2 + \left(r_0 + \rho \tanh \frac{\rho}{\rho_0}\right)^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (2)$$

or

$$ds^2 = -dt^2 + d\rho^2 + \left(r_0 + \rho \coth \frac{\rho}{\rho_0} - \rho_0\right)^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (3)$$

where  $r_0, \rho_0$  are the constants ( $r_0$  is a radius of the throat,  $\rho_0$  is the parameter which describes the length of the throat) and

$$\frac{r_0}{\rho_0} \ll 1. \quad (4)$$

The effect of self-action in the region  $\rho \lesssim \rho_0$  does not depend on the geometry of a spacetime outside of this region and we shall call this region the long throat (the accurate determination of the long throat is given below).

The organization of this Letter is as follows. In the following section we develop the general approach to a procedure of the self-force calculation. In section III, we develop an approximation for the self-force acting on a scalar charge at fixed position using the WKB approximation for the radial modes of the scalar field. In section IV, we evaluate the explicit expressions for the self-force on the two specific gravitational backgrounds. Finally, in section V we present the concluding remarks.

Throughout this Letter we use units  $c = G = 1$ .

## 2 General approach

Let us consider a massless scalar field  $\phi$  with scalar source  $j$ . The corresponding action is given by

$$S = -\frac{1}{8\pi} \int (\phi_{,\mu} \phi^{,\mu} + \xi R \phi^2) \sqrt{-g} d^4x + \int j \phi \sqrt{-g} d^4x, \quad (5)$$

where  $\xi$  is a coupling of the scalar field to the scalar curvature  $R$  and  $g$  is the determinant of the metric  $g_{\mu\nu}$ . The corresponding field equation has a form

$$\left(\square_x - \xi R(x)\right) \phi(x; \tilde{x}) = -4\pi j(x; \tilde{x}), \quad (6)$$

where

$$j(x; \tilde{x}) = q \int \delta^{(4)}(x^\mu, \tilde{x}^\mu(\tau)) \frac{d\tau}{\sqrt{-g}}, \quad (7)$$

is the scalar current,  $q$  is the scalar charge and  $\tau$  is its proper time. The world line of the charge is given by  $\tilde{x}^\mu(\tau)$ . We shall consider only the case in which the charge is at rest in an ultrastatic spacetime. This means that one can rewrite the field equation in the following way

$$\left(\Delta_x - \xi R(x^\alpha)\right) \phi(x^\alpha; \tilde{x}^\alpha) = -\frac{4\pi q}{u^t \sqrt{-g}} \delta^{(3)}(x^\alpha, \tilde{x}^\alpha), \quad (8)$$

where  $t$  is the time coordinate,  $u^t = dt/d\tau$  and  $\alpha = 1, 2, 3$ .

The procedure of the self-force evaluation requires the renormalization of a scalar potential  $\phi(x; \tilde{x})$  which is diverged in the limit  $x \rightarrow \tilde{x}$  (see, for example, [9,10]). This renormalization is achieved by subtracting from  $\phi(x; \tilde{x})$  the DeWitt-Schwinger counterterm  $\phi_{DS}(x; \tilde{x})$  and then letting  $x \rightarrow \tilde{x}$ :

$$\phi_{ren}(x) = \lim_{\tilde{x} \rightarrow x} [\phi(x; \tilde{x}) - \phi_{DS}(x; \tilde{x})]. \quad (9)$$

The expression for  $\phi_{DS}(x; \tilde{x})$  is evaluated in [12] and in 3D case has a form (see, also, [10])

$$\phi_{DS}(x; \tilde{x}) = q \frac{\Delta^{1/2}}{\sqrt{2\sigma}} \quad (10)$$

where  $\sigma$  is one-half the square of the distance between the points  $x$  and  $\tilde{x}$  along the shortest geodesic connecting them and  $\Delta$  is DeWitt-Morrett determinant.

Finally the self-force acting on a static charge is

$$f_\alpha(x) = -\frac{q}{2} \nabla_\alpha \phi_{ren}(x). \quad (11)$$

### 3 WKB approximation for the self-force

The metric of an ultrastatic spherically symmetric spacetime under consideration is

$$ds^2 = -dt^2 + d\rho^2 + r^2(\rho) (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (12)$$

In this spacetime for the static charge  $u^t = 1$  the solution of (8) can be expanded in terms of Legendre polynomials  $P_l$  with the result that

$$\phi(x^\alpha; \tilde{x}^\alpha) = q \sum_{l=0}^{\infty} (2l+1) P_l(\cos \gamma) g_l(\rho, \tilde{\rho}), \quad (13)$$

where  $\cos \gamma \equiv \cos \theta \cos \tilde{\theta} + \sin \theta \sin \tilde{\theta} \cos(\varphi - \tilde{\varphi})$  and  $g_l(\rho, \tilde{\rho})$  satisfies the equation

$$g_l'' + \frac{(r^2)'}{r^2} g_l' - \left[ \frac{l(l+1)}{r^2} + \xi R \right] g_l = -\frac{\delta(\rho, \tilde{\rho})}{r^2}. \quad (14)$$

In this expression and below a prime denotes a derivative with respect to  $\rho$ . The homogeneous solutions to this equation will be denoted by  $p_l(\rho)$  and  $q_l(\rho)$ .  $p_l(\rho)$  is chosen to be the solution which is well behaved at  $\rho = -\infty$  and divergent at  $\rho \rightarrow +\infty$ .  $q_l(\rho)$  is chosen to be the solution which is divergent at  $\rho \rightarrow -\infty$  and well behaved at  $\rho = \infty$ . Thus

$$\left\{ \frac{d}{d\rho^2} + \frac{(r^2)'}{r^2} \frac{d}{d\rho} - \left[ \frac{l(l+1)}{r^2} + \xi R \right] \right\} \begin{Bmatrix} p_l(\rho) \\ q_l(\rho) \end{Bmatrix} = 0, \quad (15)$$

$$g_l(\rho, \tilde{\rho}) = C_l p_l(\rho_{<}) q_l(\rho_{>}) = C_l \left[ \Theta(\tilde{\rho} - \rho) p_l(\rho) q_l(\tilde{\rho}) - \Theta(\rho - \tilde{\rho}) p_l(\tilde{\rho}) q_l(\rho) \right], \quad (16)$$

where  $\Theta(x)$  is the Heaviside step function, i.e.,  $\Theta(x) = 1$  for  $x > 0$  and  $\Theta(x) = 0$  for  $x < 0$ ,  $C_l$  is a normalization constant which could be absorbed into the definition of  $p_l$  and  $q_l$ . Normalization of  $g_l$  is achieved by integrating (14) once with respect to  $\rho$  from  $\tilde{\rho} - \delta$  to  $\tilde{\rho} + \delta$  and letting  $\delta \rightarrow 0$ . This results in the Wronskian condition

$$C_l \left( p_l \frac{dq_l}{d\rho} - q_l \frac{dp_l}{d\rho} \right) = -\frac{1}{r^2}. \quad (17)$$

The WKB approximation for the radial modes  $p_l$  and  $q_l$  is obtained by the change of variables [13]

$$\begin{aligned} p_l &= \frac{1}{\sqrt{2r^2 W}} \exp \left( \int^\rho W d\rho \right), \\ q_l &= \frac{1}{\sqrt{2r^2 W}} \exp \left( - \int^\rho W d\rho \right). \end{aligned} \quad (18)$$

Substitution of these expressions into (17) shows that the Wronskian condition is obeyed if

$$C_l = 1. \quad (19)$$

Substitution into the mode equation (15) gives the following equation for  $W$ :

$$W^2 = \frac{l(l+1) + 2\xi}{r^2} + \frac{(W^2)''}{4W^2} - \frac{5(W^2)'^2}{16W^4} + \frac{(r^2)''}{2r^2} - \frac{(r^2)'^2}{4r^4} + \xi \left( -2\frac{(r^2)''}{r^2} + \frac{(r^2)'^2}{2r^4} \right). \quad (20)$$

This equation can be solved iteratively when the metric function  $r^2(\rho)$  is slowly varying, that is,

$$\varepsilon_{\text{WKB}} = L_\star/L \ll 1, \quad (21)$$

where

$$L_\star(\rho) = \frac{r(\rho)}{\sqrt{2\xi}}, \quad (22)$$

and  $L$  is a characteristic scale of variation of  $r(\rho)$ :

$$\frac{1}{L(\rho)} = \max \left\{ \left| \frac{r'}{r} \right|, \left| \frac{r'}{r} \sqrt{|\xi|} \right|, \left| \frac{r''}{r} \right|^{1/2}, \left| \frac{r''}{r} |\xi| \right|^{1/2}, \left| \frac{r'''}{r} \right|^{1/3}, \left| \frac{r'''}{r} |\xi|^{3/2} \right|^{1/3}, \dots \right\}. \quad (23)$$

We shall call the region of spacetime where the metric function  $r(\rho)$  is slowly varying the long throat.

The zeroth-order WKB solution of Eq. (20) corresponds to neglecting terms with derivatives in this equation

$$W^2 = \Omega \cdot \left( 1 + O(\varepsilon_{\text{WKB}}^2) \right), \quad (24)$$

where

$$\Omega(\rho, l + 1/2) = \frac{l(l+1) + 2\xi}{r^2} = \frac{1}{r(\rho)^2} \left[ \left( l + \frac{1}{2} \right)^2 + \mu^2 \right], \quad (25)$$

and

$$\mu^2 = 2\xi - \frac{1}{4}. \quad (26)$$

Let us stress that  $\Omega$  is the exact solution of equation (20) in a spacetime with metric  $ds^2 = -dt^2 + d\rho^2 + r_0^2(d\theta^2 + \sin^2\theta d\varphi^2)$ , where  $r_0$  is constant. Below it is assumed that

$$\mu^2 > 0. \quad (27)$$

Substituting the solution (24) into (18) and (13), and neglecting terms of the second order and higher with respect to  $\varepsilon_{WKB}$  we can obtain the following expression for the zeroth-order WKB approximation for  $\phi(x^\alpha; \tilde{x}^\alpha)$  under the assumptions  $\theta = \tilde{\theta}, \varphi = \tilde{\varphi}$  and  $\tilde{\rho} = \rho + \delta\rho > \rho$

$$\phi(\rho, \theta, \varphi; \tilde{\rho}, \theta, \varphi) = \frac{q}{r(\rho)r(\tilde{\rho})} \sum_{l=0}^{\infty} \left( l + \frac{1}{2} \right) \frac{\exp \left( - \int_{\rho}^{\rho+\delta\rho} \sqrt{\Omega \left( \rho', l + \frac{1}{2} \right)} d\rho' \right)}{\sqrt[4]{\Omega \left( \rho, l + \frac{1}{2} \right) \Omega \left( \tilde{\rho}, l + \frac{1}{2} \right)}}. \quad (28)$$

The sum over  $l$  can be evaluated by using the Plana sum method (see, for example, [14])

$$\begin{aligned} \phi(\rho, \theta, \varphi; \tilde{\rho}, \theta, \varphi) = & \frac{q}{r(\rho)r(\tilde{\rho})} \lim_{\epsilon \rightarrow 0} \left\{ \int_{\epsilon}^{\infty} \frac{\exp \left( - \int_{\rho}^{\rho+\delta\rho} \sqrt{\Omega(\rho', x)} d\rho' \right)}{\sqrt[4]{\Omega(\rho, x) \Omega(\tilde{\rho}, x)}} x dx \right. \\ & + \int_{\epsilon-i\infty}^{\epsilon} \frac{\exp \left( - \int_{\rho}^{\rho+\delta\rho} \sqrt{\Omega(\rho', z)} d\rho' \right)}{\sqrt[4]{\Omega(\rho, z) \Omega(\tilde{\rho}, z)} (1 + e^{i2\pi z})} z dz \\ & \left. - \int_{\epsilon}^{\epsilon+i\infty} \frac{\exp \left( - \int_{\rho}^{\rho+\delta\rho} \sqrt{\Omega(\rho', z)} d\rho' \right)}{\sqrt[4]{\Omega(\rho, z) \Omega(\tilde{\rho}, z)} (1 + e^{-i2\pi z})} z dz \right\}. \end{aligned} \quad (29)$$

The first integral in this expression can be rewritten as follows

$$\begin{aligned} & \int_0^{\infty} \frac{\exp \left( - \int_{\rho}^{\rho+\delta\rho} \sqrt{\Omega(\rho', x)} d\rho' \right)}{\sqrt[4]{\Omega(\rho, x) \Omega(\tilde{\rho}, x)}} x dx \\ &= \sqrt{r(\rho)r(\tilde{\rho})} \int_0^{\infty} \frac{x \exp \left( - \sqrt{x^2 + \mu^2} \int_{\rho}^{\rho+\delta\rho} d\rho' / r(\rho') \right)}{\sqrt{x^2 + \mu^2}} dx \\ &= \sqrt{r(\rho)r(\tilde{\rho})} \frac{\exp \left( - \mu \int_{\rho}^{\rho+\delta\rho} d\rho' / r(\rho') \right)}{\int_{\rho}^{\rho+\delta\rho} d\rho' / r(\rho')} \end{aligned} \quad (30)$$

and expanded in powers of  $\delta\rho$

$$\begin{aligned}
& \int_0^\infty \frac{\exp\left(-\int_\rho^{\rho+\delta\rho} \sqrt{\Omega(\rho', x)} d\rho'\right)}{\sqrt[4]{\Omega(\rho, x)\Omega(\tilde{\rho}, x)}} x dx \\
&= \frac{r(\rho)^2}{\delta\rho} \left[ 1 + \left( \frac{dr(\rho)}{d\rho} - \mu \right) \frac{\delta\rho}{r(\rho)} + O(\delta\rho^2) \right].
\end{aligned} \tag{31}$$

The next two integrals in (29) do not diverge at  $\delta\rho \rightarrow 0$

$$\begin{aligned}
& \lim_{\epsilon \rightarrow 0} \left\{ \int_{\epsilon-i\infty}^\epsilon \frac{\exp\left(-\int_\rho^{\rho+\delta\rho} \sqrt{\Omega(\rho', z)} d\rho'\right)}{\sqrt[4]{\Omega(\rho, z)\Omega(\tilde{\rho}, z)} (1 + e^{i2\pi z})} z dz \right. \\
& \quad \left. - \int_\epsilon^{\epsilon+i\infty} \frac{\exp\left(-\int_\rho^{\rho+\delta\rho} \sqrt{\Omega(\rho', z)} d\rho'\right)}{\sqrt[4]{\Omega(\rho, z)\Omega(\tilde{\rho}, z)} (1 + e^{-i2\pi z})} z dz \right\} \\
&= r(\rho) \lim_{\epsilon \rightarrow 0} \left\{ \int_{i\epsilon}^{i\epsilon+\infty} \frac{x dx}{\sqrt{\mu^2 - x^2} (1 + e^{2\pi x})} \right. \\
& \quad \left. + \int_{-i\epsilon}^{-i\epsilon+\infty} \frac{x dx}{\sqrt{\mu^2 - x^2} (1 + e^{2\pi x})} + O(\delta\rho) \right\} \\
&= 2r(\rho) \int_0^\mu \frac{x dx}{\sqrt{\mu^2 - x^2} (1 + e^{2\pi x})} + O(\delta\rho).
\end{aligned} \tag{32}$$

Thus the zeroth-order WKB approximation of  $\phi$  is

$$\begin{aligned}
\phi(\rho, \theta, \varphi; \tilde{\rho}, \theta, \varphi) &= \frac{q}{\delta\rho} + \frac{q}{r(\rho)} \left( -\mu + 2 \int_0^\mu \frac{x dx}{\sqrt{\mu^2 - x^2} (1 + e^{2\pi x})} \right) \\
&\quad + O(\delta\rho).
\end{aligned} \tag{33}$$

The DeWitt-Schwinger counterterm  $\phi_{\text{ds}}(x; \tilde{x})$  in the limit  $\theta = \tilde{\theta}, \varphi = \tilde{\varphi}$  can be easily calculated using the metric (12):

$$\begin{aligned}
2\sigma &= \delta\rho^2, \Delta = 1 + O(\delta\rho^2), \\
\phi_{\text{ds}}(\rho, \theta, \varphi; \tilde{\rho}, \theta, \varphi) &= q \frac{\Delta^{1/2}}{\sqrt{2\sigma}} = q \left( \frac{1}{\delta\rho} + O(\delta\rho) \right).
\end{aligned} \tag{34}$$

Thus  $\phi_{\text{ren}}(x)$  is

$$\begin{aligned}
\phi_{\text{ren}}(x) &= \lim_{\delta\rho \rightarrow 0} [\phi(\rho, \theta, \varphi; \tilde{\rho}, \theta, \varphi) - \phi_{\text{DS}}(\rho, \theta, \varphi; \tilde{\rho}, \theta, \varphi)] \\
&= \frac{q}{r(\rho)} \left( -\sqrt{2\xi - \frac{1}{4}} + 2 \int_0^{\sqrt{2\xi - 1/4}} \frac{x dx}{(1 + e^{2\pi x}) \sqrt{2\xi - 1/4 - x^2}} \right) \\
&\quad \cdot \left( 1 + O(\varepsilon_{\text{WKB}}^2) \right),
\end{aligned} \tag{35}$$

and the single nonzero component of the self-force is

$$\begin{aligned}
f_\rho(x) &= -\frac{q}{2} \frac{\partial \phi_{\text{ren}}}{\partial \rho} = -\frac{q^2}{2r^2} \frac{dr}{d\rho} \left( \sqrt{2\xi - \frac{1}{4}} \right. \\
&\quad \left. - 2 \int_0^{\sqrt{2\xi - 1/4}} \frac{x dx}{(1 + e^{2\pi x}) \sqrt{2\xi - 1/4 - x^2}} \right) \left( 1 + O(\varepsilon_{\text{WKB}}^2) \right).
\end{aligned} \tag{36}$$

In the case  $\xi = 1/6$  we can numerically evaluate

$$F(\xi) = \sqrt{2\xi - \frac{1}{4}} - 2 \int_0^{\sqrt{2\xi - 1/4}} \frac{x dx}{(1 + e^{2\pi x}) \sqrt{2\xi - 1/4 - x^2}} \tag{37}$$

as follows  $F(1/6) \simeq 0.1723 \dots$

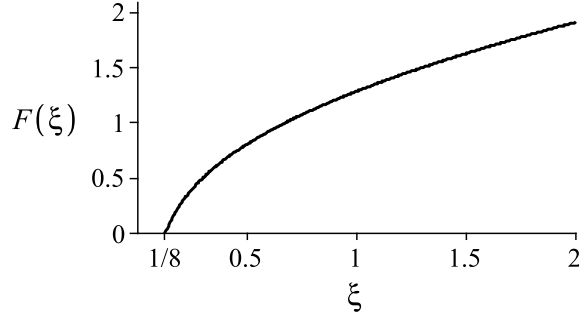


Fig. 1. The curve represents the function  $F(\xi)$ .

Let us note that if one uses  $r$  as the new radial coordinate

$$ds^2 = -dt^2 + \left( \frac{d\rho}{dr} \right)^2 dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \tag{38}$$

the expression (36) may be rewritten as follows

$$f_r = f_\rho \frac{d\rho}{dr} = -F(\xi) \frac{q^2}{2r^2} \left( 1 + O(\varepsilon_{\text{WKB}}^2) \right). \tag{39}$$



#### 4 Specific examples

First of all note that  $\varepsilon_{\text{WKB}} = 0$  in spacetime with metric

$$ds^2 = -dt^2 + d\rho^2 + r_0^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (40)$$

where  $r_0$  is constant and the expression (35) is exact. The self-force is zero in this case.

As a second example let us consider the spacetime with metric

$$ds^2 = -dt^2 + \frac{dr^2}{\left(1 - \frac{r_g}{r}\right)^n} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (41)$$

The case  $n = 2$  corresponds to the spacetime of a horn (semi-infinite throat) [15]. The part of this spacetime with  $r > r_g$  is globally static and geodesically complete. In the vicinity of  $r = r_g$

$$\left| \frac{d^m r}{r d\rho^m} \right|^{1/m} \simeq \frac{1}{r_g} \left( \frac{r - r_g}{r_g} \right)^{(n/2-1)+1/m}, \quad (42)$$

where  $\rho$  is the radial proper distance ( $dr/d\rho = (1 - r_g/r)^{n/2}$ ). Thus in the region  $r - r_g \ll r_g$

$$\varepsilon_{\text{WKB}} \simeq \left( \frac{r - r_g}{r_g} \right)^{n/2-1}. \quad (43)$$

$\varepsilon_{\text{WKB}} \ll 1$  in the case  $n > 2$  and one can call this region the long throat.

The expression (36) in the case  $r - r_g \ll r_g$  and  $n > 2$  takes the form

$$f_\rho = -F(\xi) \frac{q^2}{2r(\rho)^2} \left( 1 - \frac{r_g}{r(\rho)} \right)^{n/2} \left( 1 + O(\varepsilon_{\text{WKB}}^2) \right) \quad (44)$$

or in the coordinates (41)

$$f_r = f_\rho \frac{d\rho}{dr} = -F(\xi) \frac{q^2}{2r^2} \left( 1 + O(\varepsilon_{\text{WKB}}^2) \right). \quad (45)$$

## 5 Conclusions

The considered approach gives the possibility to compute the exact expression for the self-potential and the self-force in spacetime (40). In the long throat (12,21-23) such approach permits to obtain the approximate expression for the self-force (36, 39). Let us note that the validity of WKB approximation for all the modes (including  $l = 0$  mode) of a massless scalar field is the consequence of the nonminimal coupling ( $\xi > 1/8$ ) of a scalar field with the curvature of spacetime. This implies also that the approximate solution (24) of the equation (20) does not depend on the conditions at infinity and in considered situation the effect of self-action is the local one.

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